



TANTA UNIVERSITY  
FACULTY OF SCIENCE  
DEPARTMENT OF MATHEMATICS

EXAMINATION FOR PROSPECTIVE STUDENTS (4<sup>TH</sup> YEAR) STUDENTS OF MATHEMATICS  
COURSE TITLE: GENERAL RELATIVITY COURSE CODE: MA4113

DATE: 27/12/2016

TERM: FIRST

TOTAL ASSESSMENT MARKS:

TIME ALLOWED: 2 HOURS

Answer five questions

[1] (a) Prove that  $ds^2 = dx^2 + dy^2 + dz^2$  in parabolic coordinates  $(\xi, \eta, \varphi)$  given as  
$$ds^2 = (\xi^2 + \eta^2)d\xi^2 + (\xi^2 + \eta^2)d\eta^2 + \eta\xi d\varphi^2$$

where  $x = \xi\eta\cos\varphi$ ,  $y = \xi\eta\sin\varphi$ ,  $z = (\xi^2 - \eta^2)/2$ .

(b) When  $A_i$  is a vector, find and prove that the transformation law for

$$B_{ij} = \frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i} \text{ is tensor}$$

[2] (a) The equation  $K(ij)A_{jk} = B_{ik}$  holds for all the coordinates systems. If  $A$  and  $B$  are second-rank tensors, show that  $K$  is a second tensor also.

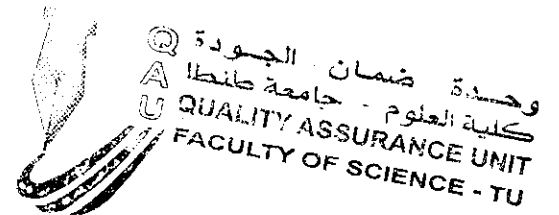
(b) Use the question [3] (b) and Obtain the equations of the geodesic for the same metric

[3] (a) Prove that  $B_{i,j}$  is tensor of rank two and Riemann-Christoffel's tensor  $B_{\mu\nu\sigma}^{\rho}$ .

(b) Find the fundamental metric and Christoffel's symbols to the metric

$$ds^2 = dt^2 - e^{2kt}(dx^2 + dy^2 + dz^2)$$

Dr. Mohamed Khalifa



|           |                             |     |
|-----------|-----------------------------|-----|
| EXAMINERS | DR./MOHAMED ABDOU KHALIFA   | DR/ |
|           | PROF./ SLIM ALI MOHAMMADEIN | DR/ |

With my best wishes

Tanta University  
 Faculty of Science  
 Mathematics Dept.  
 Year: 4  
 Pages of questions: only one page



Date: 1/1/2017  
 Time allowed: 3 h  
 Full Degree: 150

Subject: Topology 2

**Answer the following questions**

**Question 1:**

1. Prove that a continuous one-to-one function from a compact space onto a Hausdorff space is homeomorphism.
2. Show that every subset from a co-finite topology is compact.

**Question 2:**

1. Prove that every compact Hausdorff space is  $T_4$ .
2. For a continuous mapping  $f: (X, \tau_1, T_1) \rightarrow (Y, \tau_2)$ . Prove that the inverse image of points in  $Y$  is closed set in  $X$ .

**Question 3:**

1. State with proof Hein-Borel Theorem.
2. If  $X = \{a, b, c\}$  with a topology  $\tau = \{X, \emptyset, \{a\}, \{b, c\}\}$ . Discuss the regularity, normality,  $T_3$  and  $T_4$ .

**Question 4:**

1. For every topological space  $(X, \tau)$  and  $A \subset X$ , prove that  $X$  is compact iff for all  $\mathfrak{S}$  of closed subsets of  $X$  having a finite intersection property has  $\bigcap \mathfrak{S} \neq \emptyset$ .
2. Prove that the usual topology is Hausdorff.

**Question 5:**

1. Prove that a space  $X$  is  $T_1$  iff for every singleton is closed.
2. If  $X = \{a, b, c, d\}$  with a topology  $\tau = \{X, \emptyset, \{c\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ . Discuss  $T_0'$ ,  $T_1'$ ,  $T_1''$  and  $T_2''$

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| TANTA UNIVERSITY  |             | FACULTY OF SCIENCE - MATHEMATICS DEP. |                       |
| FOURTH YEAR,      |             | MA4103                                |                       |
| FLUID MECHANICS 1 |             |                                       |                       |
| DATE: 5 JAN. 2017 | TERM: FIRST | TOTAL ASSESSMENT MARKS: 150           | TIME ALLOWED: 2 HOURS |

Please answer the following questions

1A. The velocity of fluid flow in 2D is given by

$$V = yz \underline{i} + yt \underline{j} + z \underline{k}$$

through a surface of vertices (0, 5, 7), (0, 11, 7), and (0, 8, A1). If the volume flux  $Q = 918 \text{ (L}^3 \text{ T}^{-1}\text{)}$ , then calculate the constant A1, kind of surface, and average velocity.

1B. Calculate the acceleration components  $a_x$  and  $a_y$  of fluid flow by Euler and Lagrangian methods for fluid velocity

$$V = x^t \underline{i} + t^x \underline{j}$$

[45 Marks]

2A. The fluid velocity is a function of fluid density  $\rho$ , gravity  $g$ , depth  $h$ , and time  $t$ . Rewrite the relation between these parameters using Pi theory. [Hint:  $\rho$ ,  $h$ , and  $g$ , are basic parameters]

2B. Find the non-dimensional form of the following equation

$$\rho \frac{dV}{dt} = \mu \frac{d^2V}{dx^2} - \int dP - \frac{\gamma}{R}$$

and then find the non-dimensional numbers [35 Marks]

3A. If the velocity potential  $\Phi = 2xy$ , find and plot  $\Psi(x, y)$ , and path line

3B. For the 2D fluid flow, find and plot  $\Psi$  for two components velocities

$$, u = 3 + e^{-x}, v = 3 - e^y$$

[35 Marks]


4. The velocity components of fluid flow  $u=3(x^2-z^2)$ ,  $v=0$ ,  $w=-6xz$  satisfied the solution of Navier-Stokes equations. Find the pressure distribution. [Hint:  $g_x=g_y=0$ ,  $g_z=-g \underline{k}$ ] [35 Marks]

بالتوفيق والنداج

أ.د. مجدى سرواح.

لجنة الأسئلة: أ.د. سليم على محمد.

السؤال

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|  | TANTA UNIVERSITY<br>FACULTY OF SCIENCE<br>DEPARTMENT OF MATHEMATICS |                          |                    |
|   | EXAMINATION FOR COMPUTER SCIENCE (FOURTH YEAR) STUDENTS             |                          |                    |
| COURSE TITLE:   | OPERATION RES. (I)  | COURSE CODE: MA2103 4105 |                    |
| DATE: 17/1/2017   | TERM: SECOND  | TOTAL ASSESS. MARKS: 150 | TIME ALLOWED: 2 H. |

**ANSWER THE FOLLOWING QUESTION:**

[1] Find the set of all feasible solution and determine the optimal one graphically of the LP: (35 deg.)

$$LP \begin{cases} \max & z = -x_1 + 2x_2 \\ \text{s.t.} & x_1 + 2x_2 \leq 20, & 2x_1 + x_2 \leq 30, \\ & 1.5x_1 - 5x_2 \leq 15, & x_1, x_2 \geq 0. \end{cases}$$

[2] By simplex method solve of linear program: (40 deg.)

$$(LP) : \begin{cases} \min z = -5x_1 + 2x_2 - 3x_3 \\ \text{s.t.} & 2x_1 + 2x_2 - x_3 \geq 2, \\ & 3x_1 - 4x_2 \leq 3, \\ & x_2 + 3x_3 \leq 5, \\ & x_1, x_2, x_3 \geq 0 \end{cases}$$

[3] By simplex method solve of following dual linear program: (35 deg.)


$$(D) : \begin{cases} \min W = -2x_1 + 3x_2 + 5x_3 \\ \text{s.t.} & -2x_1 + 3x_2 \geq 5, \\ & -2x_1 - 4x_2 + x_3 \geq -2, \\ & x_1 + 3x_3 \geq 3, \\ & x_1, x_2, x_3 \geq 0 \end{cases}$$

[4] By minimum cost method and Vogel method solve of the transportation problem: (40 deg.)

|        | Demand |    |    |    |
|--------|--------|----|----|----|
| Supply | 4      | 7  | 13 | 9  |
| 6      | 14     | 25 | 45 | 24 |
| 8      | 60     | 25 | 39 | 50 |
| 16     | 30     | 23 | 65 | 15 |

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| EXAMINERS | PROF. E. AMMAR | GOOD LOKE |
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|  | <b>TANTA UNIVERSITY</b><br><b>FACULTY OF SCIENCE</b><br><b>DEPARTMENT OF MATHEMATICS</b>  |             |                             |                       |
|   | <b>EXAMINATION FOR PROSPECTIVE STUDENTS (4<sup>TH</sup> YEAR) STUDENTS OF MATHEMATICS</b> |             |                             |                       |
|   | <b>COURSE TITLE: QUANTUM MECHANICS II</b>   |             | <b>COURSE CODE: MA4115</b>  |                       |
| DATE: 10  | JAN 2017  | TERM: FIRST | TOTAL ASSESSMENT MARKS: 100 | TIME ALLOWED: 2 HOURS |

**Answer the following questions:**

- 1- Complete the following statements: (30 Marks)
- a) A set of vectors  $\{\Phi_n\}$  is said to linearly independent if .....
  - b) The maximum number of linearly independent vectors in a space is called .....
  - c)  $\hat{D}$  is called a linear operator if and only if ..... and .....
  - d) If  $\hat{A}$  is a Hermitian operator, then all of its eigenvalues are .....
  - e) Eigenvectors corresponding to distinct eigenvalues of a Hermitian operator must be .....
  - f) The matrix elements of the operator  $\hat{M}$  are defined as .....
  - g) The square angular momentum operator in spherical coordinates takes the form .....
  - h) The mathematical formula of the creation and annihilation operators are .....and .....
- 2- a) Use the creation and annihilation operators to find the expectation values  $\langle x \rangle$  and  $\langle x^2 \rangle$ . (15 Marks)
- b) Find the Pauli spin matrices. (15 Marks)
- c) Discuss the time independent perturbation theory. (15 Marks)
- 3- Drive the radial wave functions. (25 Marks)

|           |                     |                       |
|-----------|---------------------|-----------------------|
| EXAMINERS | PROF/AHMED ABOANBER | DR/ ABDALLAH A. NAHLA |
|           | DR/                 | DR/                   |

*With my best wishes*

السنة الأولى

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|--|------------------------------|------------------------------|
| <b>Tanta University-Faculty of Science-Department of Mathematics</b> |                              |                              |
| <b>Final Exam for the First Semester 2016-2017</b>                   |                              |                              |
| <b>Course title:</b>   | Differential geometry(1)     | <b>Course Code:</b> MA4107   |
| <b>Date:</b> 15 /1 /2017   | <b>Total mark:</b> 150 Marks | <b>Time allowed:</b> 2 Hours |

**Answer all the following questions:**

**First question: (40 Marks)**

(a)-Prove that  $\tau = - \frac{(\alpha'(s) \times \alpha''(s)) \cdot \alpha'''(s)}{k^2(s)}$

(b)-Let  $\alpha(t)$  be the parameterized curve of  $R^3$  defined by

$$\alpha(s) = \left( a \cos \frac{s}{\sqrt{a^2 + b^2}}, a \sin \frac{s}{\sqrt{a^2 + b^2}}, \frac{bs}{\sqrt{a^2 + b^2}} \right)$$

Compute  $k, \tau, t$ , and  $b$ .

**Second question: (40 Marks)**

(a)-Drive Frenet equations.

(b)-Prove that  $X(u, v)$  is a simple surface, where  $U$  is open set and

$$X(u, v) = (u, v, f(u, v))$$

(c)- Find the equation of the tangent line of the circular helix at  $t=0$ , where

$$\alpha(t) = (a \cos t, a \sin t, bt)$$

**Third question: (40 Marks)**

(a)- Show that the distance between corresponding points on two Bertrand curves is constant.

(b)- Prove that for a plane curve  $\int_0^l k(s) ds = 2\pi l$ .

(c)-Find the parameterization of the curve  $x_3^2 = 1 - x_1, x_2^2 = x_1$ .

**Fourth question: (30 Marks)**

(a)- Let  $\alpha(t) = (t, \frac{1}{2}t^2, \frac{1}{3}t^3)$ . Compute  $k, \tau$  at  $t=1$ .

(b) -Let  $\alpha: I \rightarrow R^3$  be a regular parameterized curve. Prove that  $\frac{dt}{ds} = \frac{1}{|\dot{\alpha}|} \frac{d^2 t}{ds^2} = \frac{\alpha' \cdot \alpha''}{|\dot{\alpha}|^4}$

(c) Find the curvature of the curve  $x^2 = 4ay$ .

(Best wishes)

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|-------------------|-------------------------------|----------------------|
| <b>Examiners:</b> | 1- Prof. Dr. A. E. El-Bagoury | 2- Dr. Mervat Elzawy |
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|---|--------------------------|-----------------------|
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| Date: 15 /1 /2017   | Total mark: 150 Marks    | Time allowed: 2 Hours |

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Compute  $k, \tau, t$  and  $b$ .

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**Third question: (40 Marks)**

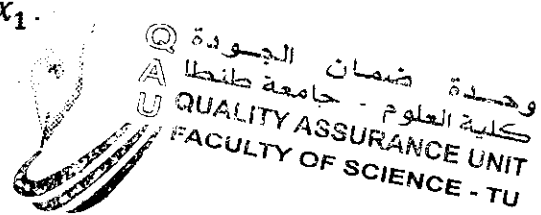
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(Best wishes)

|            |                               |                      |
|------------|-------------------------------|----------------------|
| Examiners: | 1- Prof. Dr. A. E. El-Bagoury | 2- Dr. Mervat Elzawy |
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